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PREFACE

This volume contains some reviews and original research contributions, which are related to the **7th Mathematical Physics Meeting: Summer School and Conference on Modern Mathematical Physics**, held in the Institute of Physics, Belgrade (Serbia), September 9–19, 2012 (http://www.mphys7.ipb.ac.rs). The programme of this meeting was mainly oriented towards some recent developments in gravity and cosmology, string and quantum field theory, and some relevant mathematical methods. We hope that articles presented here will be valuable literature not only for the participants of this meeting but also for many other PhD students and researchers in modern mathematical and theoretical physics. We are grateful to all authors for writing their contributions for these proceedings.

The previous six meetings in this series of summer schools and conferences on modern mathematical physics were also held in Serbia: Sokobanja, 13–25 August 2001; Kopaonik, 1–12 September 2002; Zlatibor, 20–31 August 2004; Belgrade, 3–14 September 2006; Belgrade, 6–17 July 2008; and Belgrade, 14–23 September 2010. The corresponding proceedings of all these meetings were published by the Institute of Physics, Belgrade, and are available in the printed form as well as online at the websites.

This seventh meeting was held in the Institute of Physics (Belgrade), which is at the nice bank of river Danube. There was also a sightseeing excursion by ship "Zlatno Srce" (Golden Heart) and another to TV tower hill Avala. We hope that all attending this meeting will recall it as a useful and pleasant event, and will wish to participate again in the future.

We wish to thank all lecturers and other speakers for their interesting and valuable talks. We also thank all participants for their active participation. Financial support of our sponsors: *Ministry of Education, Science and Technological Development of the Republic of Serbia, Belgrade; The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy; ICTP – SEENET–MTP grant RRJ-09 "Cosmology and Strings", Niš, Serbia; Project 174012 (Geometry, Education and Visualization with Applications), Belgrade; and Nova Škola, Belgrade,* was very significant for realization of this activity.

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Review and Research Works

Gravity, Nonlinear Gauge Fields and Charge Confinement/Deconfinement [*]

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Abstract

We discuss in some detail the properties of gravity (including $f(R)$-gravity) coupled to non-standard nonlinear gauge field system containing a square root of the usual Maxwell Lagrangian $-\frac{f_0}{2}\sqrt{-F^2}$. The latter is known to produce in flat spacetime a QCD-like confinement. Inclusion of gravity triggers various physically interesting effects: new mechanism for dynamical generation of cosmological constant; non-standard black hole solutions with constant vacuum electric field and with "hedge-hog"-type spacetime asymptotics, which are shown to obey the first law of black hole thermodynamics; new "tubelike" solutions of Levi-Civita-Bertotti-Robinson type; charge-"hiding" and charge-confining "thin-shell" wormhole solutions; dynamical effective gauge couplings and confinement-deconfinement transition effect when coupled to quadratic R^2-gravity.

1. Introduction

We consider gravity, including $f(R)$-gravity [1], coupled to *non-standard* nonlinear gauge field system containing a square root of the ordinary Maxwell Lagrangian $-\frac{f_0}{2}\sqrt{-F^2}$. In flat spacetime the latter model has been shown [2] to produce a *QCD-like confinement*.

We exhibit several interesting features of the above system (see also Refs.[3, 4]) :

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- New mechanism for *dynamical* generation of cosmological constant due to nonlinear gauge field dynamics: $\Lambda_{\text{eff}} = \Lambda_0 + 2\pi f_0^2$ (Λ_0 – bare cosmological constant, may be absent at all).
- Non-standard black hole solutions of Reissner-Nordström-(anti-)de-Sitter type containing a *constant radial vacuum electric field* (in addition to the Coulomb one), in particular, in electrically neutral black holes of Schwarzschild-(anti-)de-Sitter type. It is shown that these non-standard black holes obey the first law of black hole thermodynamics.
- In case of vanishing effective cosmological constant Λ_{eff} (*i.e.*, $\Lambda_0 < 0$, $|\Lambda_0| = 2\pi f_0^2$) the resulting Reissner-Nordström-type black hole, apart from carrying an additional constant vacuum electric field, turns out to be *non-asymptotically flat* – a feature resembling the gravitational effect of a *hedgehog* [6].
- Appearance of *confining-type effective potential* in charged test particle dynamics in the above black hole backgrounds.
- New "tubelike" solutions of Levi-Civita-Bertotti-Robinson [7] type, *i.e.*, with spacetime geometry of the form $\mathcal{M}_2 \times S^2$, where $\mathcal{M}_2$ is a two-dimensional anti-de Sitter, Rindler or de Sitter space depending on the relative strength of the electric field w.r.t. the coupling f_0 of the square-root gauge field term.

When in addition one or more *lightlike branes* are self-consistently coupled to the above gravity/nonlinear-gauge-field system (as matter and charge sources) they produce ("thin-shell") wormhole solutions displaying two novel physically interesting effects [4]:

- *"Charge-hiding" effect* - a genuinely charged matter source of gravity and electromagnetism may appear *electrically neutral* to an external observer – a phenomenon opposite to the famous Misner-Wheeler "charge without charge" effect [5];
- *Charge-confining "tubelike" wormhole* with two "throats" occupied by two oppositely charged lightlike branes – the whole electric flux is confined within the finite-extent "middle universe" of generalized Levi-Civita-Bertotti-Robinson type – no flux is escaping into the outer non-compact "universes".

Additional interesting features appear when we couple the "square-root" confining nonlinear gauge field system to $f(R)$-gravity with $f(R) = R+\alpha R^2$ and a dilaton. Reformulating the model in the physical "Einstein" frame we find (cf. second Ref.[3]):

- *"Confinement-deconfinement" transition* due to appearance of "flat" region in the effective dilaton potential;
- The effective gauge couplings as well as the induced cosmological constant become *dynamical* depending on the dilaton v.e.v. In particular, a conventional Maxwell kinetic term for the gauge field is *dynamically generated* even if absent in the original theory;

- *Regular black hole* solution (*no singularity* at $r = 0$) *with confining vacuum electric field*: the bulk spacetime consist of two regions – an interior de Sitter and an exterior Reissner-Nordström-type (with "hedgehog asymptotics") glued together along their common horizon occupied by a charged lightlike brane. The latter also dynamically determines the non-zero cosmological constant in the interior de-Sitter region. This result is analogous to the regular black hole solution in the case of ordinary Einstein gravity presented in Ref.[8] and will be discussed in more detail in a subsequent paper.

Concluding the introductory remarks, let us briefly mention the principal motivation for studying non-standard gauge field models with $\sqrt{-F^2}$. G. 't Hooft has shown [9] that in any effective quantum gauge theory, which is able to describe linear confinement phenomena, the energy density of electrostatic field configurations should be a linear function of the electric displacement field in the infrared region (the latter appearing as an "infrared counterterm").

The simplest way to realize these ideas in flat spacetime was proposed in Refs.[2]:

$$S = \int d^4x L(F^2) \quad , \quad L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2} \; , \tag{1}$$
$$F^2 \equiv F_{\mu\nu}F^{\mu\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \; ,$$

The square root of the Maxwell term naturally arises as a result of *spontaneous breakdown of scale symmetry* of the original scale-invariant Maxwell action with f_0 appearing as an integration constant responsible for the latter spontaneous breakdown. For static field configurations the model (1) yields an electric displacement field $\vec{D} = \vec{E} - \frac{f_0}{\sqrt{2}}\frac{\vec{E}}{|\vec{E}|}$ and the corresponding energy density turns out to be $\frac{1}{2}\vec{E}^2 = \frac{1}{2}|\vec{D}|^2 + \frac{f_0}{\sqrt{2}}|\vec{D}| + \frac{1}{4}f_0^2$, so that it indeed contains a term linear w.r.t. $|\vec{D}|$. The model (1) produces, when coupled to quantized fermions, a confining effective potential $V(r) = -\frac{\beta}{r} + \gamma r$ (Coulomb plus linear one with $\gamma \sim f_0$) which is of the form of the well-known "Cornell" potential in the phenomenological description of quarkonium systems in QCD [10].

2. Einstein Gravity Coupled to Confining Nonlinear Gauge Field

The pertinent action is given by (R-scalar curvature; Λ_0 - bare cosmological constant, might be absent):

$$S = \int d^4x \sqrt{-G}\Big[\frac{R - 2\Lambda_0}{16\pi} + L(F^2)\Big] \; , \; L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2} \; , \tag{2}$$
$$F^2 \equiv F_{\kappa\lambda}F_{\mu\nu}G^{\kappa\mu}G^{\lambda\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \; .$$

Remark. One could start with the non-Abelian version of the gauge field action in (2). Since we will be interested in static spherically symmetric solutions, the non-Abelian gauge theory effectively reduces to an Abelian one.

The corresponding equations of motion read accordingly – Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R + \Lambda_0 G_{\mu\nu} = 8\pi T^{(F)}_{\mu\nu} \; , \tag{3}$$

$$T^{(F)}_{\mu\nu} = \Bigl(1 - \frac{f_0}{\sqrt{-F^2}}\Bigr) F_{\mu\kappa}F_{\nu\lambda}G^{\kappa\lambda} - \frac{1}{4}\Bigl(F^2 + 2f_0\sqrt{-F^2}\Bigr)G_{\mu\nu} \; , \tag{4}$$

and nonlinear gauge field equations:

$$\partial_\nu \left(\sqrt{-G}\Bigl(1 - \frac{f_0}{\sqrt{-F^2}}\Bigr) F_{\kappa\lambda}G^{\mu\kappa}G^{\nu\lambda}\right) = 0 \; . \tag{5}$$

Important remark. Note the *non-zero* value of the trace of energy-momentum tensor unlike ordinary Maxwell theory:

$$T^{(F)} \equiv T^{(F)}_{\mu\nu}G^{\mu\nu} = -f_0\sqrt{-F^2} \; .$$

Solving Eqs.(3)–(5) we find new *non-standard* Reissner-Nordström-(anti-) de-Sitter-type black holes depending on the sign of a dynamically generated cosmological constant $\Lambda_{\rm eff}$:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \; , \tag{6}$$

$$A(r) = 1 - \sqrt{8\pi}|Q|f_0 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda_{\rm eff}}{3}r^2 \quad , \quad \Lambda_{\rm eff} = 2\pi f_0^2 + \Lambda_0 \; , \tag{7}$$

with static spherically symmetric electric field containing apart from the Coulomb term an additional *constant* “vacuum” piece:

$$F_{0r} = \frac{\varepsilon_F f_0}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi}\, r^2} \quad , \quad \varepsilon_F \equiv \mathrm{sign}(F_{0r}) = \mathrm{sign}(Q) \; . \tag{8}$$

The latter corresponds to a confining “Cornell”-type [10] potential $A_0 = -\frac{\varepsilon_F f_0}{\sqrt{2}}\, r + \frac{Q}{\sqrt{4\pi}\, r}$. When $\Lambda_{\rm eff} = 0$, $A(r) \to 1 - \sqrt{8\pi}|Q|f_0$ for $r \to \infty$, *i.e.*, the black hole exhibits “hedgehog” [6] *non-flat-spacetime* asymptotics.

Furthermore, we find three distinct types of static solutions of “tube-like” Levi-Civita-Bertotti-Robinson [7] type with spacetime geometry of the form $\mathcal{M}_2 \times S^2$, where $\mathcal{M}_2$ is some 2-dimensional manifold ((anti-)de Sitter $(A)dS_2$, Rindler $Rind_2$):

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + r_0^2(d\theta^2 + \sin^2\theta d\varphi^2) \; , \; -\infty < \eta < \infty \; , \tag{9}$$

$$F_{0\eta} = c_F = \mathrm{const} \quad , \quad \frac{1}{r_0^2} = 4\pi c_F^2 + \Lambda_0 \; (= \mathrm{const}) \; . \tag{10}$$

(i) $AdS_2 \times S^2$ with constant vacuum electric field $|F_{0\eta}| \equiv |\vec{E}| = |c_F|$:

$$A(\eta) = 4\pi \left[c_F^2 - \sqrt{2} f_0 |c_F| - \frac{\Lambda_0}{4\pi} \right] \eta^2 \quad (\eta - \text{Poincare patch coordinate}) \, , \tag{11}$$

provided either $|c_F| > \frac{f_0}{\sqrt{2}} \Big(1 + \sqrt{1 + \frac{\Lambda_0}{2\pi f_0^2}} \Big)$ for $\Lambda_0 \geq -2\pi f_0^2$ or $|c_F| > \sqrt{\frac{1}{4\pi} |\Lambda_0|}$ for $\Lambda_0 < 0$, $|\Lambda_0| > 2\pi f_0^2$.

(ii) $Rind_2 \times S^2$ with constant vacuum electric field $|F_{0\eta}| = |c_F|$, where $Rind_2$ is the flat 2-dimensional Rindler spacetime with:

$$A(\eta) = \eta \;\text{ for } 0 < \eta < \infty \quad \text{or} \quad A(\eta) = -\eta \;\text{ for } -\infty < \eta < 0 \tag{12}$$

provided $|c_F| = \frac{f_0}{\sqrt{2}} \Big(1 + \sqrt{1 + \frac{\Lambda_0}{2\pi f_0^2}} \Big)$ for $\Lambda_0 > -2\pi f_0^2$.

(iii) $dS_2 \times S^2$ with weak const vacuum electric field $|F_{0\eta}| = |c_F|$, where dS_2 is the 2-dimensional de Sitter space with:

$$A(\eta) = 1 - 4\pi \left[\sqrt{2} f_0 |c_F| - c_F^2 + \frac{\Lambda_0}{4\pi} \right]) \, \eta^2 \, , \tag{13}$$

when $|c_F| < \frac{f_0}{\sqrt{2}} \Big(1 + \sqrt{1 + \frac{\Lambda_0}{2\pi f_0^2}} \Big)$ for $\Lambda_0 > -2\pi f_0^2$. Note that dS_2 has *two horizons* at $\eta = \pm \eta_0 \equiv \pm \Big[4\pi \left(\sqrt{2} f_0 |c_F| - c_F^2 \right) + \Lambda_0 \Big]^{-\frac{1}{2}}$.

3. Bulk Gravity/Nonlinear Gauge Field Coupled to Lightlike Brane Sources

In the following two Sections we will consider bulk Einstein/non-linear gauge field system (2) self-consistently coupled to $N \geq 1$ (distantly separated) charged codimension-one *lightlike* p-brane (*LL-brane*) sources (here $p = 2$).

World-volume *LL-brane* actions in a reparametrization-invariant Nambu-Goto-type or in an equivalent Polyakov-type formulation were proposed in Refs.[11]:

$$S_{\text{LL}}[q] = -\frac{1}{2} \int d^{p+1}\sigma \, T b_0^{\frac{p-1}{2}} \sqrt{-\gamma} \left[\gamma^{ab} \bar{g}_{ab} - b_0 (p-1) \right] \, , \tag{14}$$

$$\bar{g}_{ab} \equiv \partial_a X^\mu G_{\mu\nu} \partial_b X^\nu - \frac{1}{T^2} (\partial_a u + q \mathcal{A}_a)(\partial_b u + q \mathcal{A}_b) \;\; , \;\; \mathcal{A}_a \equiv \partial_a X^\mu A_\mu \, . \tag{15}$$

Here and below the following notations are used:

- γ_{ab} is the *intrinsic* world-volume Riemannian metric; $g_{ab} = \partial_a X^{\mu} G_{\mu\nu}(X) \partial_b X^{\nu}$ is the *induced* metric on the world-volume, which becomes *singular* on-shell (manifestation of the lightlike nature); b_0 is world-volume "cosmological constant".
- $X^{\mu}(\sigma)$ are the p-brane embedding coordinates in the bulk D-dimensional spacetime with Riemannian metric $G_{\mu\nu}(x)$ $(\mu, \nu = 0, 1, \ldots, D-1)$; $(\sigma) \equiv \left(\sigma^0 \equiv \tau, \sigma^i\right)$ with $i = 1, \ldots, p$; $\partial_a \equiv \frac{\partial}{\partial \sigma^a}$.
- u is auxiliary world-volume scalar field defining the lightlike direction of the induced metric;
- T is *dynamical (variable)* brane tension;
- q – the coupling to bulk spacetime gauge field $\mathcal{A}_{\mu}$ is *LL-brane* surface charge density.

The on-shell singularity of the induced metric g_{ab} , *i.e.*, the lightlike property, directly follows from the *LL-brane* equations of motion:

$$g_{ab} \left(\bar{g}^{bc}(\partial_c u + q\mathcal{A}_c)\right) = 0 \; . \tag{16}$$

Now, let us consider the full action of self-consistently coupled bulk Einstein/non-linear gauge field/*LL-brane* system $(L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2})$:

$$S = \int d^4x \sqrt{-G} \Big[\frac{R(G) - 2\Lambda_0}{16\pi} + L(F^2) \Big] + \sum_{k=1}^{N} S_{\mathrm{LL}}[q^{(k)}] \; , \tag{17}$$

where the superscript (k) indicates the k-th *LL-brane*.

The corresponding equations of motion are as follows:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R + \Lambda_0 G_{\mu\nu} = 8\pi \Big[T^{(F)}_{\mu\nu} + \sum_{k=1}^{N} T^{(k)}_{\mu\nu} \Big] \; , \tag{18}$$

$$\partial_{\nu} \Big[\sqrt{-G} \Big(1 - \frac{f_0}{\sqrt{-F^2}} \Big) F_{\kappa\lambda} G^{\mu\kappa} G^{\nu\lambda} \Big] + \sum_{k=1}^{N} j^{\mu}_{(k)} = 0 \; . \tag{19}$$

The energy-momentum tensor and the charge current density of k-th *LL-brane* are straightforwardly derived from the pertinent *LL-brane* world-volume action (14):

$$T^{\mu\nu}_{(k)} = - \int d^3\sigma \, \frac{\delta^{(4)}\Big(x - X_{(k)}(\sigma)\Big)}{\sqrt{-G}} \, T^{(k)} \sqrt{|\bar{g}_{(k)}|} \bar{g}^{ab}_{(k)} \partial_a X^{\mu}_{(k)} \partial_b X^{\nu}_{(k)} \; , \tag{20}$$

$$j^{\mu}_{(k)} = -q^{(k)} \int d^3\sigma \, \delta^{(4)}\Big(x - X_{(k)}(\sigma)\Big) \sqrt{|\bar{g}_{(k)}|} \bar{g}^{ab}_{(k)} \partial_a X^{\mu}_{(k)} \frac{\partial_b u^{(k)} + q^{(k)} \mathcal{A}^{(k)}_b}{T^{(k)}} \; . \tag{21}$$

Solving Eqs.(18)–(19) with (20)–(21) we find "thin-shell" wormhole solutions of static "spherically-symmetric" type (in Eddington-Finkelstein coordinates $dt = dv - \frac{d\eta}{A(\eta)}$, $F_{0\eta} = F_{v\eta}$):

$$ds^2 = -A(\eta)dv^2 + 2dvd\eta + C(\eta)h_{ij}(\theta)d\theta^i d\theta^j \, , \qquad F_{v\eta} = F_{v\eta}(\eta) \, , \tag{22}$$

$$-\infty < \eta < \infty \qquad A(\eta_0^{(k)}) = 0 \ \text{ for } \ \eta_0^{(1)} < \ldots < \eta_0^{(N)} \, . \tag{23}$$

The derivation of these "thin-shell" wormhole solutions proceeds along the following main steps:

(i) Take "vacuum" solutions of (18)–(19) (without delta-function *LL-brane* terms) in each spacetime region (separate "universe") given by $(-\infty < \eta < \eta_0^{(1)}), \ldots, (\eta_0^{(N)} < \eta < \infty)$ with common horizon(s) at $\eta = \eta_0^{(k)}$ $(k = 1, \ldots, N)$.

(ii) Each k-th *LL-brane* automatically locates itself on the horizon at $\eta = \eta_0^{(k)}$ – intrinsic property of *LL-brane* dynamics [11].

(iii) Match discontinuities of the derivatives of the metric and the gauge field strength across each horizon at $\eta = \eta_0^{(k)}$ using the explicit expressions for the *LL-brane* stress-energy tensor and charge current density (20)–(21).

4. Charge "Hiding"and Charge Confining Wormholes

First we will construct "one-throat" wormhole solutions to (17) with the charged *LL-brane* occupying the wormhole "throat", which connects (i) a non-compact "universe" with Reissner-Nordström-(anti)-de-Sitter-type geometry (where the cosmological constant is partially or entirely *dynamically* generated) to (ii) a compactified ("tubelike") "universe" of (generalized) Levi-Civita-Bertotti-Robinson type with geometry $AdS_2 \times S^2$ or $Rind_2 \times S^2$.

These wormholes possess the novel property of *hiding* electric charge from external observer in the non-compact "universe". Namely, the whole electric flux produced by the charged *LL-brane* at the wormhole "throat" is pushed into the "tubelike" "universe". As a result, the non-compact "universe" becomes electrically neutral with Schwarzschild-(anti-)de-Sitter or purely Schwarzschild geometry. Therefore, an external observer in the non-compact "universe" detects a *genuinely charged* matter source (the charged *LL-brane*) as *electrically neutral.*

The explicit form $ds^2 = -A(\eta)dv^2 + 2dvd\eta + C(\eta)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$ for the metric and the nonlinear gauge theory's electric field $F_{v\eta}(\eta)$ read:

- "Left universe" of Levi-Civita-Bertotti-Robinson ("tubelike") type with geometry $AdS_2 \times S^2$ for $\eta < 0$:

$$A(\eta) = 4\pi\Bigl(c_F^2 - \sqrt{2}f_0|c_F| - \frac{\Lambda_0}{4\pi}\Bigr)\eta^2 \; , \; C(\eta) \equiv r_0^2 = \frac{1}{4\pi c_F^2 + \Lambda_0} \; , \qquad (24)$$

$$|F_{v\eta}| \equiv |\vec{E}| = |c_F| > \frac{f}{\sqrt{2}}\Bigl(1 + \sqrt{1 + \frac{\Lambda_0}{2\pi f_0^2}}\Bigr) \quad \text{for } \Lambda_0 > -2\pi f_0^2 \; ,$$

$$\text{or} \quad |F_{v\eta}| \equiv |\vec{E}| = |c_F| > \sqrt{\frac{1}{4\pi}|\Lambda_0|} \quad \text{for } \Lambda_0 < 0 \; , \; |\Lambda_0| > 2\pi f_0^2 \; .$$

- Non-compact "right universe" for $\eta > 0$ comprising the exterior region of Reissner-Nordström-de-Sitter-type black hole beyond the middle (Schwarzschild-type) horizon r_0 when $\Lambda_0 > -2\pi f_0^2$ (in particular, when $\Lambda_0 = 0$), or the exterior region of Reissner-Nordström-*anti*-de-Sitter-type black hole beyond the outer (Schwarzschild-type) horizon r_0 in the case $\Lambda_0 < 0$ and $|\Lambda_0| > 2\pi f_0^2$, or the exterior region of Reissner-Nordström-"hedgehog" black hole for $|\Lambda_0| = 2\pi f_0^2$ (note: $A(\eta) \equiv A_{\mathrm{RN-((A)dS)}}(r_0 + \eta)$):

$$A(\eta) = 1 - \sqrt{8\pi}|Q|f_0 - \frac{2m}{r_0+\eta} + \frac{Q^2}{(r_0+\eta)^2} - \frac{\Lambda_0 + 2\pi f_0^2}{3}(r_0+\eta)^2 \; , \qquad (25)$$

$$C(\eta) = (r_0+\eta)^2 \quad , \quad |F_{v\eta}| \equiv |\vec{E}| = \frac{f_0}{\sqrt{2}} + \frac{|Q|}{\sqrt{4\pi}\,(r_0+\eta)^2} \; .$$

The matching relations for the discontinuities of the metric and gauge field components across the *LL-brane* world-volume occupying the wormhole "throat" (which are here derived self-consistently from a well-defined world-volume Lagrangian action principle for the *LL-brane*) (14) determine all parameters of the wormhole solutions as functions of q (the *LL-brane* charge) and f_0 (coupling constant of $\sqrt{-F^2}$):

$$Q = 0 \quad , \quad |c_F| = |q| + \frac{f_0}{\sqrt{2}} \; , \qquad (26)$$

as well as the allowed range for the "bare" cosmological constant:

$$-4\pi\Bigl(|q| + \frac{f_0}{\sqrt{2}}\Bigr)^2 < \Lambda_0 < 4\pi\Bigl(q^2 - \frac{f_0^2}{2}\Bigr) \; , \qquad (27)$$

The relations (26) (recall $|F_{v\eta}| \equiv |\vec{E}| = |c_F|$ in the "tubelike" "left universe") have profound consequences:

(A) The non-compact "right universe" (25) becomes exterior region of electrically neutral Schwarzschild-(*anti*-)de-Sitter or purely Schwarzschild black

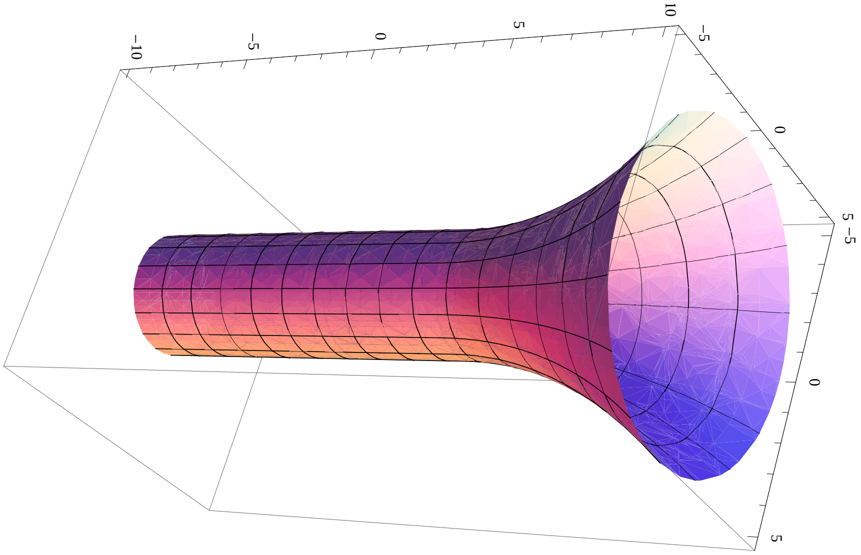

Figure 1: Shape of $t = const$ and $\theta = \frac{\pi}{2}$ slice of *charge-"hiding"* wormhole geometry: the whole electric flux produced by the charged *LL-brane* at the "throat" is expelled into the left infinitely long cylindric tube.

hole beyond the Schwarzschild horizon carrying a vacuum constant radial electric field $|F_{v\eta}| \equiv |\vec{E}| = \frac{f_0}{\sqrt{2}}$.

(B) Recalling that the dielectric displacement field is $\vec{D} = \left(1 - \frac{f_0}{\sqrt{2}|\vec{E}|}\right)\vec{E}$, we find from the second relation (26) that the whole flux produced by the charged *LL-brane* flows only into the "tubelike" "left universe" (24) (since $\vec{D} = 0$ in the non-compact "right universe"). This is a novel property of *hiding electric charge through a wormhole* connecting non-compact to a "tubelike" universe from external observer in the non-compact "universe".

The *charge-"hiding"* wormhole geometry is visualized on Fig.1 below.

Further, we find more interesting "two-throat" wormhole solution exhibiting *QCD-like charge confinement* effect – obtained from a self-consistent coupling of the gravity/nonlinear-gauge-field system with two identical *oppositely charged LL-branes* (Eq.(17) with $N = 2$). The total "two-throat" wormhole spacetime manifold is made of:

(i) "Left-most" non-compact "universe" comprising the exterior region of Reissner-Nordström-de-Sitter-type black hole beyond the middle Schwarzschild-type horizon r_0 for the "radial-like" η-coordinate interval:

$$-\infty < \eta < -\eta_0 \equiv -\left[4\pi\left(\sqrt{2}f_0|c_F| - c_F^2\right) + \Lambda_0\right]^{-\frac{1}{2}} , \qquad (28)$$

where:

$$\begin{aligned} A(\eta) = A_{\rm RNdS}(r_0 - \eta_0 - \eta) &= 1 - \sqrt{8\pi}|Q|f_0 - \frac{2m}{r_0 - \eta_0 - \eta} \\ &+ \frac{Q^2}{(r_0 - \eta_0 - \eta)^2} - \frac{\Lambda_0 + 2\pi f_0^2}{3}(r_0 - \eta_0 - \eta)^2 \;, \qquad (29) \\ C(\eta) = (r_0 - \eta_0 - \eta)^2 \;\; , \;\; |F_{v\eta}(\eta)| &\equiv |\vec{E}| = \frac{f_0}{\sqrt{2}} + \frac{|Q|}{\sqrt{4\pi}\,(r_0 - \eta_0 - \eta)^2} \;. \end{aligned}$$

(ii) "Middle" "tube-like" "universe" of Levi-Civita-Bertotti-Robinson type with geometry $dS_2 \times S^2$ comprising the finite extent (w.r.t. η-coordinate) region between the two horizons of dS_2 at $\eta = \pm\eta_0$:

$$-\,\eta_0 < \eta < \eta_0 \equiv \Bigl[4\pi\left(\sqrt{2}f_0|c_F| - c_F^2\right) + \Lambda_0\Bigr]^{-\frac{1}{2}} \;, \qquad (30)$$

where the metric coefficients and electric field are:

$$\begin{aligned} &A(\eta) = 1 - \Bigl[4\pi\left(\sqrt{2}f_0|c_F| - c_F^2\right) + \Lambda_0\Bigr]\,\eta^2 \;\; , \;\; A(\pm\eta_0) = 0\,, \\ &\hfill (31) \\ &C(\eta) = r_0^2 = \frac{1}{4\pi c_F^2 + \Lambda_0} \;\; , \quad |F_{v\eta}| \equiv |\vec{E}| = |c_F| < \frac{f}{\sqrt{2}}\Bigl(1 + \sqrt{1 + \frac{\Lambda}{2\pi f_0^2}}\Bigr), \end{aligned}$$

with $\Lambda_0 > -2\pi f_0^2$;

(iii) "Right-most" non-compact "universe" comprising the exterior region of Reissner-Nordström-de-Sitter-type black hole beyond the middle Schwarzschild-type horizon r_0 for the "radial-like" η-coordinate interval $\eta_0 < \eta < \infty$ (η_0 as in (30)), where:

$$\begin{aligned} A(\eta) = A_{\rm RNdS}(r_0 + \eta - \eta_0) &= 1 - \sqrt{8\pi}|Q|f_0 - \frac{2m}{r_0 + \eta - \eta_0} \\ &+ \frac{Q^2}{(r_0 + \eta - \eta_0)^2} - \frac{\Lambda_0 + 2\pi f_0^2}{3}(r_0 + \eta - \eta_0)^2 \;, \qquad (32) \\ C(\eta) = (r_0 + \eta - \eta_0)^2 \;\; , \;\; |F_{v\eta}(\eta)| &\equiv |\vec{E}| = \frac{f_0}{\sqrt{2}} + \frac{|Q|}{\sqrt{4\pi}\,(r_0 + \eta - \eta_0)^2} \;. \end{aligned}$$

As dictated by the *LL-brane* dynamics [11] each of the two *LL-branes* locates itself on one of the two common horizons at $\eta = \pm\eta_0$ between "left" and "middle", and between "middle" and "right" "universes", respectively.

The matching relations for the discontinuities of the metric and gauge field components across the each of the two *LL-brane* world-volumes determine all parameters of the wormhole solutions as functions of $\pm q$ (the opposite

LL-brane charges) and f_0 (coupling constant of $\sqrt{-F^2}$). Most importantly we obtain:

$$Q = 0 \quad , \quad |c_F| = |q| + \frac{f_0}{\sqrt{2}} \; , \tag{33}$$

and the bare cosmological constant must be in the interval:

$$\Lambda_0 \leq 0 \quad , \quad |\Lambda_0| < 2\pi(f_0^2 - 2q^2) \quad \to \quad |q| < \frac{f_0}{\sqrt{2}} \; , \tag{34}$$

in particular, Λ_0 could be zero.

Similarly to the charge-“hiding” case, relations (33) meaning:

$$|\vec{E}|_{\text{middle universe}} = |q| + |\vec{E}|_{\text{left/right universe}} \; ,$$

have profound consequences:

- The “left-most” (29) and “right-most” (32) non-compact “universes” become two identical copies of the *electrically neutral* exterior region of Schwarzschild-de-Sitter black hole beyond the Schwarzschild horizon. They both carry a constant vacuum radial electric field with magnitude $|\vec{E}| = \frac{f_0}{\sqrt{2}}$ pointing inbound towards the horizon in one of these “universes” and pointing outbound w.r.t. the horizon in the second “universe”. The corresponding electric displacement field $\vec{D} = 0$, so there is *no* electric flux there (recall $\vec{D} = \left(1 - \frac{f_0}{\sqrt{2}|\vec{E}|}\right)\vec{E}$).
- The whole electric flux produced by the two charged *LL-branes* with opposite charges $\pm q$ at the boundaries of the above non-compact “universes” is *confined* within the “tube-like” middle “universe” (31) of Levi-Civita-Robinson-Bertotti type with geometry $dS_2 \times S^2$, where the constant electric field is $|\vec{E}| = \frac{f_0}{\sqrt{2}} + |q|$ with associated non-zero electric displacement field $|\vec{D}| = |q|$. This is *QCD-like confinement*.

A simple visualization of the *charge-confining* wormhole geometry is given in Fig.2.

5. R^2-Gravity Coupled to Confining Nonlinear Gauge Field and Dilaton

Consider now coupling of $f(R) = R + \alpha R^2$ gravity (possibly with a bare cosmological constant Λ_0) to a “dilaton” ϕ and the nonlinear gauge field

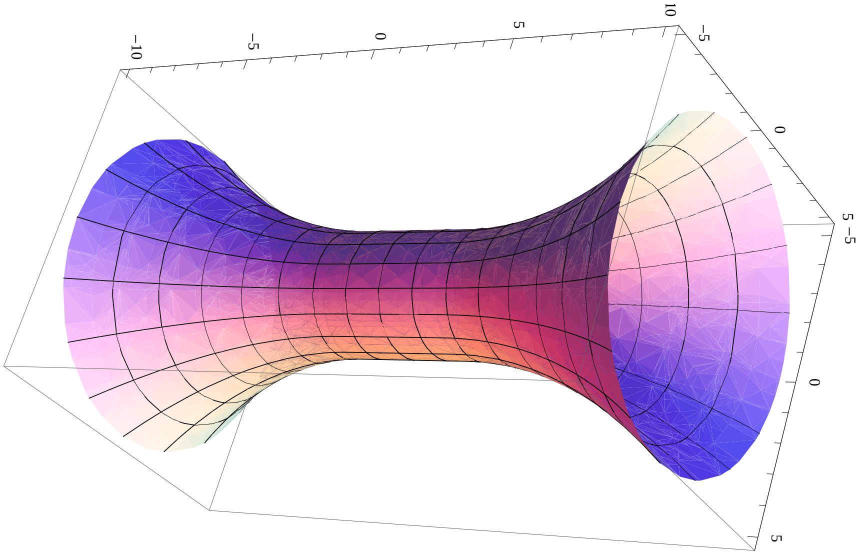

Figure 2: Shape of $t = const$ and $\theta = \frac{\pi}{2}$ slice of *charge-confining* wormhole geometry: the whole electric flux produced by the two oppositely charged *LL-branes* is confined within the middle finite-extent cylindric tube between the "throats".

system containing $\sqrt{-F^2}$:

$$S = \int d^4x \sqrt{-g} \Big[\frac{1}{16\pi} \Big(f(R(g,\Gamma)) - 2\Lambda_0 \Big) + L(F^2(g)) + L_D(\phi, g) \Big], \tag{35}$$

$$f(R(g,\Gamma)) = R(g,\Gamma) + \alpha R^2(g,\Gamma) \quad , \quad R(g,\Gamma) = R_{\mu\nu}(\Gamma) g^{\mu\nu} \, , \tag{36}$$

$$L(F^2(g)) = - \frac{1}{4e^2} F^2(g) - \frac{f_0}{2} \sqrt{-F^2(g)} \, , \tag{37}$$

$$F^2(g) \equiv F_{\kappa\lambda} F_{\mu\nu} g^{\kappa\mu} g^{\lambda\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \, , \tag{38}$$

$$L_D(\phi, g) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi). \tag{39}$$

$R_{\mu\nu}(\Gamma)$ is the Ricci curvature in the first order (Palatini) formalism, *i.e.*, the spacetime metric $g_{\mu\nu}$ and the affine connection $\Gamma^{\mu}_{\nu\lambda}$ are *a priori* independent variables.

The equations of motion resulting from the action (35) read:

$$R_{\mu\nu}(\Gamma) = \frac{1}{f'_R} \left[8\pi T_{\mu\nu} + \frac{1}{2} f(R(g,\Gamma)) g_{\mu\nu} \right] \quad , \quad f'_R \equiv \frac{df(R)}{dR} = 1 + 2\alpha R(g,\Gamma) \, , \tag{40}$$

$$\nabla_\lambda \left(\sqrt{-g}\, f'_R g^{\mu\nu}\right) = 0 \; , \tag{41}$$

$$\partial_\nu \Big(\sqrt{-g}\Big[1/e^2 - \frac{f_0}{\sqrt{-F^2(g)}}\Big] F_{\kappa\lambda} g^{\mu\kappa} g^{\nu\lambda}\Big) = 0 \; . \tag{42}$$

The total energy-momentum tensor is given by:

$$\begin{aligned} T_{\mu\nu} = &\Big[L(F^2(g)) + L_D(\phi,g) - \frac{1}{8\pi}\Lambda_0\Big] g_{\mu\nu} \\ &+ \Big(1/e^2 - \frac{f_0}{\sqrt{-F^2(g)}}\Big) F_{\mu\kappa} F_{\nu\lambda} g^{\kappa\lambda} + \partial_\mu \phi \partial_\nu \phi \; . \end{aligned} \tag{43}$$

Eq.(41) leads to the relation $\nabla_\lambda \left(f'_R g_{\mu\nu}\right) = 0$ and thus it implies transition to the "physical" Einstein-frame metrics $h_{\mu\nu}$ via conformal rescaling of the original metric $g_{\mu\nu}$ [12]:

$$g_{\mu\nu} = \frac{1}{f'_R} h_{\mu\nu} \quad , \quad \Gamma^\mu_{\nu\lambda} = \frac{1}{2} h^{\mu\kappa} \left(\partial_\nu h_{\lambda\kappa} + \partial_\lambda h_{\nu\kappa} - \partial_\kappa h_{\nu\lambda}\right) \; . \tag{44}$$

Using (44) the R^2-gravity equations of motion (40) can be rewritten in the form of *standard* Einstein equations:

$$R^\mu_\nu(h) = 8\pi \left(T_{\rm eff}{}^\mu_\nu(h) - \frac{1}{2}\delta^\mu_\nu T_{\rm eff}{}^\lambda_\lambda(h) \right) \tag{45}$$

with effective energy-momentum tensor of the following form:

$$T_{{\rm eff}\,\mu\nu}(h) = h_{\mu\nu} L_{\rm eff}(h) - 2\frac{\partial L_{\rm eff}}{\partial h^{\mu\nu}} \; . \tag{46}$$

The effective Einstein-frame matter Lagrangian reads (the dilaton kinetic term $X(\phi,h) \equiv -\frac{1}{2} h^{\mu\nu} \partial_\mu \phi \partial_n \phi$ will be ignored in the sequel):

$$\begin{aligned} L_{\rm eff}(h) = &-\frac{1}{4e^2_{\rm eff}(\phi)} F^2(h) - \frac{1}{2} f_{\rm eff}(\phi) \sqrt{-F^2(h)} \\ &+ \frac{X(\phi,h)(1+16\pi\alpha X(\phi,h)) - V(\phi) - \Lambda_0/8\pi}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} \end{aligned} \tag{47}$$

with the following dynamical ϕ-dependent couplings:

$$\frac{1}{e^2_{\rm eff}(\phi)} = \frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} \; , \tag{48}$$

$$f_{\rm eff}(\phi) = f_0 \frac{1 + 32\pi\alpha X(\phi,h)}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} \; . \tag{49}$$

Thus, all equations of motion of the original R^2-gravity system (35)–(39) can be equivalently derived from the following Einstein/nonlinear-gauge-field/dilaton action:

$$S_{\rm eff} = \int d^4x \sqrt{-h} \Bigl[\frac{R(h)}{16\pi} + L_{\rm eff}(h) \Bigr] \; , \tag{50}$$

where $R(h)$ is the standard Ricci scalar of the metric $h_{\mu\nu}$ and $L_{\rm eff}(h)$ is as in (47).

Important observation. Even if ordinary kinetic Maxwell term $-\frac{1}{4}F^2$ is absent in the original system ($e^2 \to \infty$ in (37)), such term is nevertheless *dynamically generated* in the Einstein-frame action (47)–(50), which is a *combined effect* of αR^2 and $-\frac{f_0}{2}\sqrt{-F^2}$:

$$S_{\rm maxwell} = -4\pi\alpha f_0^2 \int d^4x \sqrt{-h} \frac{F_{\kappa\lambda}F_{\mu\nu}h^{\kappa\mu}h^{\lambda\nu}}{1+8\alpha\,(8\pi V(\phi)+\Lambda_0)} \; . \tag{51}$$

In what follows we consider constant "dilaton" ϕ extremizing the effective Lagrangian (47):

$$L_{\rm eff} = -\frac{1}{4e^2_{\rm eff}(\phi)} F^2(h) - \frac{1}{2} f_{\rm eff}(\phi)\sqrt{-F^2(h)} - V_{\rm eff}(\phi), \tag{52}$$

$$V_{\rm eff}(\phi) = \frac{V(\phi)+\frac{\Lambda_0}{8\pi}}{1+8\alpha\,(8\pi V(\phi)+\Lambda_0)} \; , \; f_{\rm eff}(\phi) = \frac{f_0}{1+8\alpha\,(8\pi V(\phi)+\Lambda_0)} \; , \tag{53}$$

$$\frac{1}{e^2_{\rm eff}(\phi)} = \frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1+8\alpha\,(8\pi V(\phi)+\Lambda_0)} \; . \tag{54}$$

Important observation. The dynamical couplings and effective potential are extremized *simultaneously* – this is an explicit realization of "least coupling principle" of Damour-Polyakov [13]:

$$\frac{\partial f_{\rm eff}}{\partial \phi} = -64\pi\alpha f_0 \frac{\partial V_{\rm eff}}{\partial \phi} \;\; , \;\; \frac{\partial}{\partial \phi}\frac{1}{e^2_{\rm eff}} = -(32\pi\alpha f_0)^2 \frac{\partial V_{\rm eff}}{\partial \phi} \;\; \to \frac{\partial L_{\rm eff}}{\partial \phi} \sim \frac{\partial V_{\rm eff}}{\partial \phi} \; . \tag{55}$$

Therefore at the extremum of $L_{\rm eff}$ (52) ϕ must satisfy:

$$\frac{\partial V_{\rm eff}}{\partial \phi} = \frac{V'(\phi)}{\left[1+8\alpha\,(\kappa^2 V(\phi)+\Lambda_0)\right]^2} = 0 \; . \tag{56}$$

There are two generic cases:

(a) *Confining phase*: Eq.(56) is satisfied for some finite-value ϕ_0 extremizing the original potential $V(\phi)$: $V'(\phi_0) = 0$.

(b) *Deconfinement phase*: For polynomial or exponentially growing original $V(\phi)$, so that $V(\phi) \to \infty$ when $\phi \to \infty$, we have:

$$\frac{\partial V_{\rm eff}}{\partial \phi} \to 0 \quad , \quad V_{\rm eff}(\phi) \to \frac{1}{64\pi\alpha} = {\rm const} \quad \text{when} \;\; \phi \to \infty \; , \tag{57}$$

i.e., for sufficiently large values of ϕ we find a "flat region" in $V_{\rm eff}$. This "flat region" triggers a *transition from confining to deconfinement dynamics.*

Namely, in the "flat-region" case ($V(\phi) \to \infty$) we have from (53)–(54):

$$f_{\rm eff} \to 0 \quad , \quad e^2_{\rm eff} \to e^2 \tag{58}$$

and the effective gauge field Lagrangian (52) reduces to the ordinary *non-confining* one (the "square-root" term $\sqrt{-F^2}$ vanishes):

$$L^{(0)}_{\rm eff} = -\frac{1}{4e^2} F^2(h) - \frac{1}{64\pi\alpha} \tag{59}$$

with an *induced* cosmological constant $\Lambda_{\rm eff} = 1/8\alpha$, which is *completely independent* of the bare cosmological constant Λ_0.

Within the physical "Einstein"-frame in the confining phase ($V'(\phi_0) = 0$, $\phi_0 =$ finite) we find:

(A) Reissner-Nordström-(*anti-*)de-Sitter type black holes, in particular, non-standard Reissner-Nordström type with non-flat "hedgehog" asymptotics, generalizing solutions (6)–(8) in the ordinary Einstein-gravity case, where now the effective cosmological constant and the vacuum constant radial electric field read:

$$\Lambda_{\rm eff}(\phi_0) = \frac{\Lambda_0 + 8\pi V(\phi_0) + 2\pi e^2 f_0^2}{1 + 8\alpha \left(\Lambda_0 + 8\pi V(\phi_0) + 2\pi e^2 f_0^2\right)} \; , \tag{60}$$

$$|\vec{E}_{\rm vac}| = \Bigl(\frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha \left(8\pi V(\phi_0) + \Lambda_0\right)}\Bigr)^{-1} \frac{f_0/\sqrt{2}}{1 + 8\alpha \left(8\pi V(\phi_0) + \Lambda_0\right)} \; . \tag{61}$$

(B) Levi-Civita-Bertotti-Robinson type "tubelike" spacetimes with geometries $AdS_2 \times S^2$, $Rind_2 \times S^2$ and $dS_2 \times S^2$ generalizing (9)–(13), where now (using short-hand notation $\Lambda(\phi_0) \equiv 8\pi V(\phi_0) + \Lambda_0$):

$$\frac{1}{r_0^2} = \frac{4\pi}{1 + 8\alpha\Lambda(\phi_0)} \Bigl[\Bigl(1 + 8\alpha \left(\Lambda(\phi_0) + 2\pi f_0^2\right)\Bigr)\vec{E}^2 + \frac{1}{4\pi}\Lambda(\phi_0)\Bigr] \; . \tag{62}$$

6. Discussion

Inclusion of the non-standard nonlinear "square-root" gauge field term provides explicit realization of the old "classic" idea of 't Hooft [9] about the nature of low-energy confinement dynamics. Coupling of nonlinear gauge theory containing $\sqrt{-F^2}$ to gravity (Einstein or $f(R) = R + \alpha R^2$ plus scalar "dilaton") leads to a variety of remarkable effects:

- Dynamical effective gauge couplings and dynamical induced cosmological constant;
- New non-standard black hole solutions of Reissner-Nordström-(*anti*-)de-Sitter type carrying an additional constant vacuum electric field, in particular, non-standard Reissner-Nordström type black holes with asymptotically non-flat "hedgehog" [6] behavior;
- "Cornell"-type [10] confining potential in charged test particle dynamics;
- Coupling to a charged lightlike brane produces a charge-"hiding" wormhole, where a genuinely charged matter source is detected as electrically neutral by an external observer;
- Coupling to two oppositely charged lightlike brane sources produces a two-"throat" wormhole displaying a genuine QCD-like charge confinement.
- When coupled to $f(R) = R + \alpha R^2$ gravity plus scalar "dilaton", the $\sqrt{-F^2}$ term triggers a transition from confining to deconfinement phase. Standard Maxwell kinetic term for the gauge field is dynamically generated even when absent in the original "bare" theory. The above are cumulative effects produced by the *simultaneous* presence of αR^2 and $\sqrt{-F^2}$ terms.

Let us conclude with a brief remark concerning the thermodynamic properties of the non-standard black hole solutions described above. To this end, let us recall that for any static spherically symmetric metric of the form (6) with Schwarzschild-type horizon r_0, *i.e.*, $A(r_0) = 0$, $\partial_r A\big|_{r_0} > 0$, the so called *surface gravity* κ proportional to Hawking temperature T_h (*e.g.* [14], Ch. 12.5) is given by $\kappa = 2\pi T_h = \frac{1}{2}\partial_r A\big|_{r_0}$. With $A(r)$ of the general form $A(r) = 1 - c(Q_i) - 2m/r + A_1(r;Q_i)$, where Q_i are the rest of the black hole parameters apart from the mass m, and $c(Q_i)$ is generically a non-zero constant as in (7) (responsible for the "hedgehog" non-flat spacetime asymptotics), one can straightforwardly derive the first law of black hole thermodynamics for the above class of solutions:

$$\delta m = \frac{1}{8\pi}\kappa\delta A_H + \widetilde{\Phi}_i \delta Q_i\,, \quad A_H = 4\pi r_0^2\,,\ \widetilde{\Phi}_i = \frac{r_0}{2}\frac{\partial}{\partial Q_i}\Bigl(A_1(r_0;Q_i) - c(Q_i)\Bigr)\,. \tag{63}$$

In the special case of non-standard Reissner-Nordström-(anti-)de-Sitter type black holes (6)–(7) with parameters (m,Q) the conjugate potential in (63):

$$\widetilde{\Phi} = \sqrt{4\pi}\Bigl(\frac{Q}{\sqrt{4\pi}r_0} - \frac{f_0}{\sqrt{2}}r_0\Bigr) = \sqrt{4\pi}A_0\,\big|_{r=r_0} \tag{64}$$

is (up to a constant factor) the electric field potential of the nonlinear gauge system on the horizon.

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